Introduction and General Outline:

Symmetry (similarity) analysis is quite interesting and a very sophisticated subject of research for applied mathematicians. This is because; similarity is concerned with invariance of an aspect of a given problem under continuous symmetries. When a problem is invariant under a continuous transformation, i.e., it has a continuous symmetry, then often the complexity in solving it is reduced by the dimension of the symmetry. Thus For a given problem, often one can determine admitted symmetries and conservation laws that are not obvious by inspection. In turn, one can use admitted symmetries and conservation laws for intermediate or long-time asymptotic analysis, as an aid for determining numerical solutions, to obtain particular solutions, to map the problem to a simpler problem (e.g., linearization), etc.

In the most general sense, symmetry can be defined as a property that preserves some of its aspects under certain actual or possible transformation. For example: A sphere is symmetrical because a rotation about its axis preserves its shape. The general concept of symmetry applies not only to objects and their collection, but also to properties of object, to processes they may undergo, as well as to more abstract entities such as mathematical structures, scientific laws and symbolic and conceptual systems including mythology and religion.

In the recent year there have been considerable developments in application of group symmetry techniques for the simplification of differential equations. This is because, if differential equation (linear/nonlinear, Ordinary/partial) when found Symmetric (invariant) under certain group of transformation it yields amazing results. While we do not have the mathematical tools to develop the full range of symmetry techniques, we will learn how to exploit some of the most basic symmetry properties: translations, leading to traveling wave solutions, scalings, leading to similarity solutions, rotational symmetries.

A symmetry group of a system of differential equations is a group of transformation that maps any solution to another solution of the system. In general, by symmetry of an equation, we mean a transformation that takes solutions to solutions. Thus, knowing a symmetry transformation, if we are in possession of one solution, then we can construct a second solution by applying the symmetry and possibly, a third solution by applying the symmetry yet again and so on. If we have many symmetries, then we can produce many solutions by this simple device.

Symmetry techniques for differential equations are originally developed by Sophus Lie. Lie (1881) has shown that such transformations are groups, So called Lie group, that depend on continuous parameters and consist of point transformation, contact transformations acting on the space of independent and dependent variables or independent and dependent variables as well as on all first derivatives of the dependent variables.

The key idea of Lie's theory of symmetry analysis of differential equations relies on the invariance of the latter under a transformation of independent and dependent variables. This transformation forms a local group of point transformation, establishing a diffeomorphism on the space of independent and dependent variables, mapping solutions of the equations to other solutions. Any transformation of the independent and dependent variables in turn induces a transformation of the derivatives. Lie's theory is powerful, versatile, and fundamental to the development of systematic procedures leading to the integration by quadrature (or at least to lowering the order) of ordinary differential equations, to the determination of invariant solutions of initial and boundary value problems, to the derivation of conserved quantities, or to the construction of relations between different differential equations that turn out to be equivalent.

If a system of PDEs is invariant under a Lie group of transformations, one can find, constructively, special solutions, called similarity solutions or invariant solutions that are invariant under a subgroup of the full group admitted by the system. A similarity solution of a partial differential equation is one that remains unchanged (invariant) under a one-parameter family of symmetries. These solutions result from solving a reduced system of differential equations with fewer independent variables. Invariant solutions can also be constructed for specific boundary value problems. Here one seeks a subgroup of the full group of a given PDE that leaves invariant the boundary curves and the conditions imposed on them [Bluman and Cole (1974)]. Such solutions include self-similar (auto model) solutions that can be obtained through dimensional analysis or, more generally, from invariance under groups of transformation.

The method to reduce the number of independent variable/s from the PDE's /system of PDE's so as to reduce it in to ODE's /system of ODE's is known as similarity method. Thus similarity transformation reduces the number of independent variables in the partial differential equations.

Today the various similarity transformations are available in literature. These techniques can be classified into two groups, namely direct methods and group theoretic methods. The direct methods of similarity analysis are those which do not invoke group symmetry (invariance) and they are fairly straight forward and simple to apply. The separations of variables method, the free parameter method, the dimensional analysis method [due to Sedov (1959)] and the generalized dimensional analysis method [due to Moran (1971)] are the examples of direct methods. There is no doubt that the application of these methods is valuable in special cases. But the group invariance methods can be applied in a more straightforward manner and are more general. These Group methods are not based on linear operator, superposition and other requirement of linear solution technique and also the group invariance criteria is quit advance topic of research. In this work, they are applicable to transform partial differential equation into ordinary differential equation, to transform boundary value problem into initial value problem.

Our main interest is in the symmetries inherent in the physical laws that govern the natural world. Knowledge of these symmetries will be used to enhance our understanding of complex physical phenomena, to simplify and solve problems, and, ultimately, to deepen our understanding of nature. The primary goal of this present research work is to develop the general group theoretic technique to determine similarity transformations for non-linear partial differential equations governing motion of non-Newtonian flow past different external bodies. We mainly deal with the systematic derivation of similarity transformations using the basic concept of modern group theory.

Present Investigation: -

We have systematically developed a general group transformation method using S. Lie's continuous point/contact transformation groups that enables one to search for all possible conditions under which:

1. The given non-linear partial differential equations of boundary value type can be transformed into ordinary differential equations. (Similarity Method)

2. We have used our general group transformation method to transform the homogeneous boundary conditions into nonhomogeneous boundary conditions and further the transformation

of boundary value problems into initial value problems. The type of general group transformation used for such applications are not found in literature.

The study will be done on such problems, which have wide applications in day-to-day life as well as in industry. The entire research work is presented in the SIX well planned chapters. The brief discussion of each chapter is given below.

Chapter-1 is introductory chapter which contains the theoretical background along with some survey of past research work that needed for the entire subsequent chapters to form the basis for the entire thesis. Root of the symmetry techniques along with some important terminologies, definitions and statements of important theorems are discussed in detail in this chapter.

Chapter -2 is devoted to survey of the available methods for the determination of similarity transformations. The systematic general group theoretic method is developed in this chapter. This method along with possible modification and extension is applied to various physical problems in the subsequent chapters to reduce the number of independent variables by more than one in a single step. Actually, the reduction of the independent variable is a simplification of the original mathematical description whether the resulting description is solved analytically, numerically or by using other approximate procedures whatsoever is found suitable.

In chapter-3, first we have given introduction of non-Newtonian fluids and classification of these fluids with respect to their empirical (stress-strain) relationships. The general group theoretic similarity analysis discussed in chapter-2 is applied to derive similarity solution for the system of non-linear partial differential equations governing the boundary layer flow of laminar incompressible heat and mass transfer of convective viscous non-Newtonian fluid flow past semi-infinite inclined surface with chemical reaction. The systems of non-linear partial differential equations are reduced to system of non-linear ordinary differential equations with appropriate boundary conditions. [**Newtonian case of this chapter published under the title "Group theoretic technique for natural convection heat and mass transfer in an inclined surface with chemical reaction", *Int.J.of Appl. Math and Mach (IJAMM),* 8(5), 28-39, (2011).]**

Chapter 4, contains systematic development of general group theoretic analysis to transform boundary value problem into initial value problem. This type of transformation technique is quite useful for the simplification of both linear and non-linear ordinary differential equations of boundary value type governing different physical situation. As an example, the method applied to the equations of Rayleigh problem for a non-Newtonian MHD Sisko fluid without porous media and Rayleigh Problem for a MHD Sisko fluid with porous media where certain physical parameter appears in the differential equation. The numerical solutions of the problems under consideration are also discussed. The results are plotted in graph form. It shows that, velocity of the fluid flow decrease as the value of η increase. **[The part of this chapter with title: "Group Theoretic Analysis of Rayleigh Problem for a Non-Newtonian MHD Sisko Fluid Past Semi Infinite Plate" is communicated for publication to *International Journal Of Advances In Applied Mathematics And Mechanics*.]**

Chapter 5, we introduced physical and mathematical background of jet flow. Further we have derived a combined non-liner partial differential equation governing motion of both two dimensional and axisymmetric jets. The generalized group symmetry method which discussed in chapter 2 is applied to found combined similarity equation for both two dimensional and axisymmetric jet flows of non-Newtonian power law fluids. Thus first we have transformed system of partial differential equations into system of ordinary differential equations. The two dimensional jet and axisymmetric jet that have homogeneous boundary conditions at second point is also analyzed in such a way that first of all homogeneous boundary condition at second point is transformed into non-homogeneous form and then boundary value is transformed into initial values (known as missing initial condition). Thus here one parameter general group of transformation applied to transform homogeneous boundary condition into non homogeneous form. Then, again introduce a new one parameter general group of transformation to transform boundary value problem to initial value problem. The method of transformation of homogeneous boundary condition into non-homogeneous form and the transformation of entire boundary value problem into initial value problem by our general group transformation technique is a novel technique and not found in literature. The numerical results for the two dimensional and axisymmetric jet flows are calculate and compared with those available in literature. **[Part of this chapter published as: "Group Theoretic Method For Two Dimensional Jet Of**

Incompressible Viscous Fluids", *Int. e-journal for education and mathematics*, 1(1), pp 15-26, (2012).]

In chapter-6, the group theoretic technique is applied to the system of equations governing the unsteady boundary layer flow of non Newtonian micro-polar fluid near rear a porous medium the partial differential equation governing the present flow situation is highly non linear along with suitable auxiliary conditions. This system of partial differential equation contains three independent variables and hence to reduce it into system of ordinary differential equation with appropriate boundary condition one has to apply general multi parameter (two parameter) group of transformation. In this chapter, the group theoretic technique given in chapter two is extended and applied to present flow equations to convert it into the system of ordinary differential equations with appropriate boundary conditions. **[Part of this chapter published under the title: "Similarity Solution For Unsteady Boundary Layer Flow of a Micropolar Non-Newtonian Power Law Fluid Through A Porous Medium", *Int. J. of mathematical Archive*, 3(1), pp 143-152, (2012).]**

It is still thought that much work is left to be done in the area of application of group symmetry methods to problems of fluid mechanics. But, the aim of the present study is to provide qualitative work on the successful development of general one and two parameter group-theoretic transformations along with the application of the theory to some non-Newtonian flow problems. The technique is quit promising and further extension is wide open for more complicated real life problem of various fields, like, bio fluid mechanics, magneto hydrodynamics, medical science etc.

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Research Guide

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